Effect of Recycle Structure on Distillation Tower Time Constants

Recycle of material or energy can change the effective dynamic behavior of a process. Distillation towers are inherently recycle processes. Published tower time constants based on linear analysis have been substantially in error when compared to actual responses for many cases. The reason for this error is shown to be the recycle nature of distillation towers. An improved method of estimating time constants, which can be extended to industrial towers, is presented.

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Distillation is the most commonly used separation process in the chemical and petroleum industries. This paper treats the subject of approximate modeling of distillation tower dynamics. Such models are useful for feedforward control and operator training. In addition, some companies use approximate models to decide operability questions such as whether or not surge tankage is needed on the feed to a tower and what types of control schemes to use.

Linearization techniques have been used to estimate approximate tower time constants. For high-purity towers the resulting estimates are known to be substantially in error when compared to either actual or simulated tower responses. In this paper it is shown that the recycle structure of distillation towers is the reason that the linearized estimates are in error. It is further demonstrated that substantially better estimates of tower dynamics can be obtained if a perturbed steady state rather than the design steady state is used in a linearization analysis for high-purity towers.

CONCLUSIONS AND SIGNIFICANCE

The results presented in this paper are significant for a number of reasons. First, the essential reason why distillation tower time constants obtained via linearization can be substantially in error, especially for high-purity towers, is explained. The error results from a positive feedback loop produced by the recycle tower structure. Near the normal design conditions the gain of this loop can be close to (but always less than) 1.0. Positive feedback systems with such loop gains respond as if they had very large time constants. The second significant finding is that for distillation towers the gain of the positive feedback loop drops sharply for small perturbations away from steady state. As a result it can be concluded that for reasonable disturbances a perturbed steady state rather than the design steady state should be used to estimate tower dynamic response. Using this approach on six towers previously treated by other authors produced significantly better estimates of the dominant tower time constants for the highest-purity cases. A number of towers with a variety of characteristics have been examined. The key conclusion about using a perturbed steady state appears valid for all towers. The work presented here is one of the few studies that discusses the relationship between the structure of a process and its approximate dynamic model. It gives fundamental insights into the nature of distillation tower dynamics and presents an analysis method that can be applied to a wide variety of physical systems. The approach presented is easily extended to real industrial towers. Lastly, the key insights on the dynamic behavior of towers can be determined from steady state calculations.

Introduction

An important research area where additional insights are needed is in the relationship between process structure and process dynamics. Marlin (1981) pointed out that chemical processes are built up from a few basic structures, and he gave a number of practical examples of each. The focus of this present paper is on recycle structures and how they affect distillation tower dynamics. Gilliland et al. (1964) were among the first to study the effect of recycle structures on process dynamics. Subsequent to their work, Denn and co-workers published several papers on the subject (Attir and Denn, 1978; Denn, 1979; Denn and Lavie, 1982). Lastly, the work of Rinard (1982) can be noted. A key conclusion resulting from all of these past studies is that the response time of recycle processes can be substantially longer than the response time of the forward path alone. Another key conclusion is that the gain of recycle processes and therefore the process steady state sensitivity can be substantially larger than the gain of the forward path alone. Both of these effects result from the positive feedback nature of recycle processes.

Distillation towers are inherently recycle processes. In this paper the effect of this recycle nature on estimating approximate tower time constants is discussed. Wahl and Harriott (1970) used a linearized tower model and published charts that could be used to estimate tower time constants. To use these charts one needs only steady state information. Thurston (1981) has discussed how one industrial company uses Wahl and Harriott's results to configure tower control systems. Fuentes and Luyben (1983) also have published tower time constants obtained from a linearized model. For one of these cases a simulated tower responded in about three hours, while the time constant estimated from a linearized model was over 2,800 hours!

In this paper it is shown that the major contributor to the error in estimating tower time constants is the recycle structure of distillation columns. In developing this result fundamental insights into the nature of distillation column dynamics are obtained. An improved method of estimating tower time constants is presented. This approach uses a model that is linearized around a perturbed steady state rather than the design steady state. Substantial improvements in time constant estimates are achieved when compared to detailed computer simulations.

Simple Recycle Model

To explain the effect of recycle on process dynamics the system shown in Figure 1a is briefly analyzed. The results of this analysis are applied to distillation towers in the next section. The transfer function relating $\bar{y}$ to $\bar{x}$ is

$$\frac{\bar{y}}{\bar{x}} = \frac{K_F/(T_F s + 1)}{1 - K_F K_R/[(T_F s + 1)(T_R s + 1)]} \tag{1}$$

Suppose that one wishes to approximate the structure shown in Figure 1a by an equivalent time constant system, Figure 1b. In this case $\bar{x}$ and $\bar{y}$ are given as

$$\frac{\bar{y}}{\bar{x}} = \frac{K}{\theta s + 1} \tag{2}$$

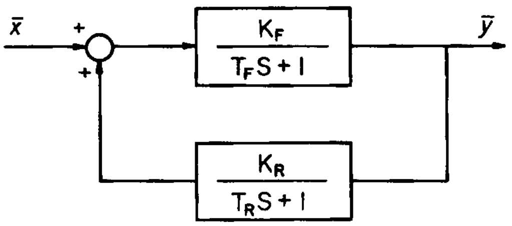

Figure 1a. Block diagram of a recycle structure.

There are several ways of relating K and θ to K_F, K_R, T_F and T_R. The method of moments (Gibilaro and Lees, 1969) will be used, as proposed by Marlin (1983). Equating the zeroth and first moments of the impulse response of Eqs. 1 and 2 gives

$$K = \frac{K_F}{1 - K_F K_R} \tag{3}$$

$$\theta = \frac{T_F + K_F K_R T_R}{1 - K_F K_R} \tag{4}$$

Figure 2 shows a comparison of Eqs. 1 and 2 for the case where $T_F = T_R$, $K_F = 1$, and K_R takes on several values. Figure 2 shows that the transient time is significantly longer than one would expect from the size of T_F and T_R as $K_R \rightarrow 1$. It can also be seen that the moment approximation is very accurate in this case.

Equations 3 and 4 point out three important features of recycle processes that earlier workers have noted (Gilliland, 1964; Attir and Denn, 1979; Denn and Lavie, 1982; Rinard 1982; Marlin, 1983). First, for $0 < K_F K_R < 1$ the steady state gain is increased relative to the gain of the forward path, K_F. Second, for $K_F K_R > 0$ the effective time constant θ is increased relative to T_F. As the product $K_F K_R \rightarrow 1$, both of these effects become more pronounced because the same term appears in both denominators. Third, for loop gains close to 1 the ratio of K/θ is essentially independent of $K_F K_R$. For step forcing K/θ is the initial rate of response of y away from steady state. Thus, even though θ is large and the recycle system takes a long time to reach steady state, the initial rate of response to step forcing is independent of θ. If y rather than y/K were plotted in Figure 2, the essentially constant initial slope would be evident. Although they are considerably more complex, distillation columns have the same type of positive feedback structure as the system shown in Figure 1. The positive feedback structure results from the recycle nature of distillation columns. For distillation columns the loop gain, $K_F K_R$, is always less than 1. However, in high-purity towers the loop gain approaches 1 at design conditions, and this is the reason for the linearized time constants becoming so large.

Distillation Tower Time Constants

To explain the effect of recycle structure on distillation tower time constant estimates, six towers discussed by Fuentes and Luyben (1983) are considered since they cover a representative

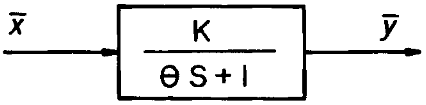

Figure 1b. An equivalent time constant system for the recycle structure shown in Fig. 1a.

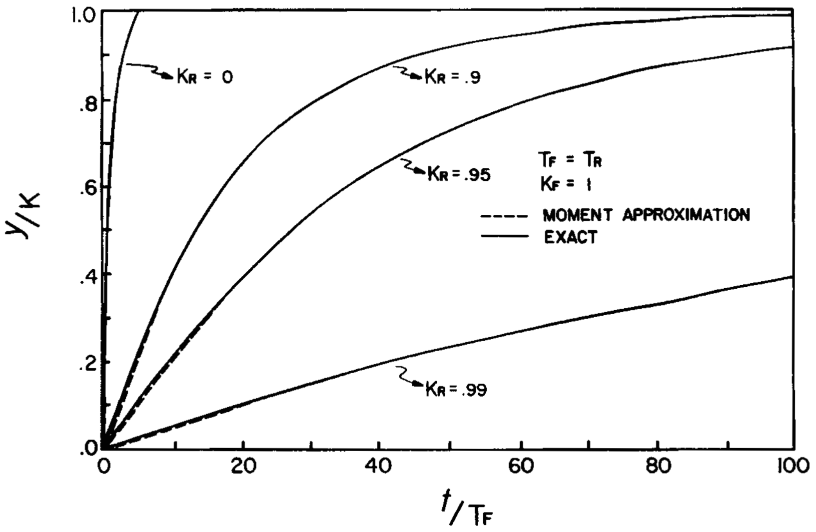

Figure 2. Comparison of Eqs. 1 and 2.

sample of two-product towers. The essential features of these towers are shown in Table 1.

The model used by Fuentes and Luyben assumed constant molal overflow and constant relative volatility. They linearized the model and used a stepping technique to calculate frequency responses numerically. The corner frequencies were determined and from them the time constants shown in Table 1 were calculated. These time constants relate changes in x_D to changes in x_F at constant product and internal tower flows. Both the linearized model and the stepping technique are discussed by Luyben (1973). Figure 3 is taken from Fuentes and Luyben's paper; it shows the response of the nonlinear model to feed composition changes. As can be seen, the highest purity towers respond much faster than the linearized time constants would indicate. A simplified dynamic model can be developed to determine the reason for these discrepancies.

In Figure 4 a schematic diagram of a distillation tower is shown. To simplify the presentation the feed is assumed to be a saturated liquid. The essential results obtained are independent of this assumption. Since flow rates are constant for the results shown in Figure 3, they will be taken as constant in the derivation that follows. For each block in Figure 4 the inputs are the total moles of light component flowing in (flow × mole fraction), and the outputs are the total moles of the light component flowing out. The various transfer functions that can be defined are shown in Figure 5. Since the gain of all the G_i's is positive, Figure 5 indicates that there are a number of positive feedback loops in a tower's dynamic structure. The transfer functions relating $\overline{Dx_D}$ and $\overline{Bx_B}$ to $\overline{Fx_F}$ can be derived from Figure 5 as follows. First the transfer functions relating $\overline{Lx_{NE}}$ to $\overline{Vy_{NE+1}}$ and $\overline{Vy_{NE+1}}$ to $\overline{L'x'}$ are determined from standard block diagram algebra as

$$\frac{\overline{Lx_{NE}}}{\overline{Vy_{NE+1}}} = G_7 + \frac{G_8G_9G_{11}}{1 - G_{10}G_{11}} \equiv H_E \tag{5}$$

$$\frac{\overline{Vy_{NE+1}}}{\overline{L'x'}} = G_1 + \frac{G_2G_3G_5}{1 - G_4G_5} \equiv H_S \tag{6}$$

$$\frac{\overline{x_D D}}{\overline{Vy_{NE+1}}} = \frac{G_9G_{12}}{1 - G_{10}G_{11}} \tag{7}$$

where the subscripts E and S refer to the enriching and stripping sections. Next a material balance on the feed mixing junction gives

$$\overline{Fx_F} + \overline{Lx_{NE}} = \overline{L'x'} \tag{8}$$

Elimination of $\overline{Lx_{NE}}$, $\overline{Vy_{NE+1}}$, and $\overline{L'x'}$ in Eqs. 5–8 gives

$$\frac{\overline{Dx_D}}{\overline{Fx_F}} = \frac{H_s}{(1 - H_EH_S)}\,\frac{G_9G_{12}}{(1 - G_{10}G_{11})} \tag{9}$$

Similarly, the transfer function relating $\overline{Bx_B}$ to $\overline{Fx_F}$ can be derived as

$$\frac{\overline{Bx_B}}{\overline{Fx_F}} = \frac{1}{(1 - H_EH_S)}\,\frac{G_2G_6}{(1 - G_4G_5)} \tag{10}$$

Since flows are taken as constant they can be factored from the lefthand side of Eqs. 9 and 10 to determine transfer functions relating product composition changes to feed composition changes. The $1 - H_EH_S$ terms in Eqs. 9 and 10, the $1 - G_{10}G_{11}$ term in Eqs. 5 and 9, and the $1 - G_4G_5$ term in Eqs. 6 and 10 indicate positive feedback loops resulting from the recycle structure. Actually, there are more than three positive feedback loops in a tower; in fact, each tray involves a positive feedback loop.

Table 1. Properties of Towers Studied (x_F = 0.5)

Case	Feed kmol/min	α	$(1 - x_D) = x_B$	N_T	N_S	L/D	θ min
1	65.8	4	0.05	13	7	0.533	10
2	61.7	4	0.001	26	13	0.737	450
3	61.7	4	0.00001	40	20	0.780	45,400
4	36.3	2	0.05	18	9	2.11	30
5	31.8	2	0.001	40	20	2.43	1,630
6	31.8	2	0.00001	60	30	2.67	170,000

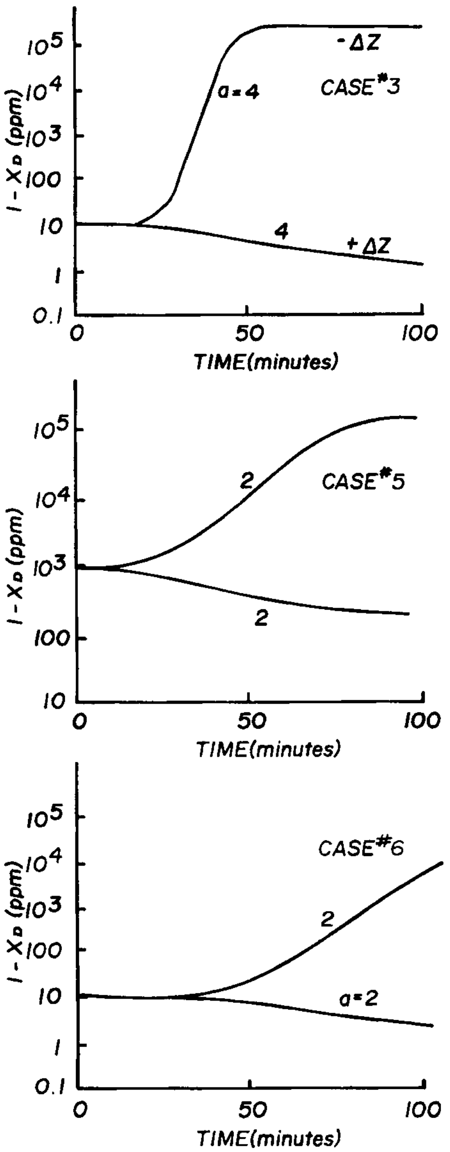

Figure 3. Nonlinear simulation of distillate composition responses to feed composition disturbances.
(a) Case 3. (b) Case 5. (c) Case 6.

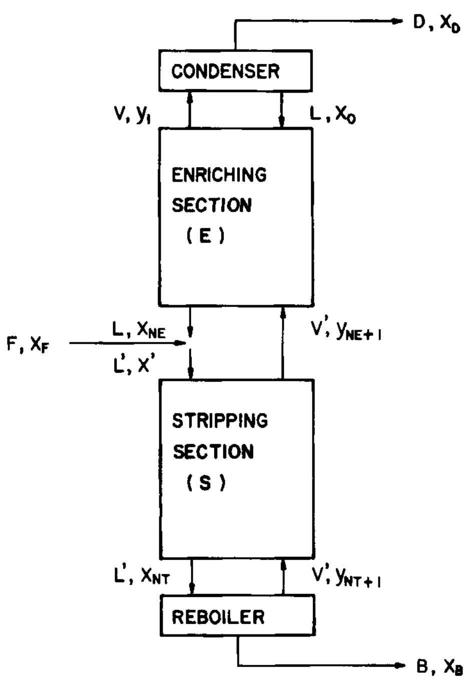

Figure 4. Schematic diagram of a distillation column.

However, by lumping the two sections of a tower into H_E and H_S, it is possible to elucidate the underlying reason for the large time constants that result from a linear analysis. Suppose that one estimates tower time constants by matching the moments of Eqs. 9 and 10 to Eq. 2. If the loop gain of any of the positive feedback loops is very close to 1.0, then the resulting approximate time constant (θ) and gain (K) will both be large. In Table 2 the gains of the G_4G_5, $G_{10}G_{11}$, and H_EH_S terms are tabulated for the six towers treated in Table 1.

These gains were calculated from Fuentes and Luyben's model using the stepping technique. As can be seen for cases 3, 5, and 6 the gain of H_EH_S is close to 1.0. These three cases are those where the time constant estimates given in Table 1 are in error by a substantial amount. These three cases also involve towers with very high purity products. The proximity of the H_EH_S gains to 1.0 indicates that a linearized approach to estimating tower time constants, e.g., matching moments or a frequency response approach, will yield extremely large values. If the H_EH_S gain were close to 1.0 over the entire range of tower operation, then the tower response would indeed be extremely sluggish. However, the H_EH_S gain changes sharply with the tower operating conditions. This fact can be used to suggest a much more accurate approach to estimating tower time constants, discussed in the following section.

The G_4G_5 and $G_{10}G_{11}$ positive feedback loops will also give rise to larger time constants. In the Appendix bounds on the G_4G_5 and $G_{10}G_{11}$ gains are derived. It is shown that for most towers the gain of these loops does not approach 1.0 as closely as the H_EH_S

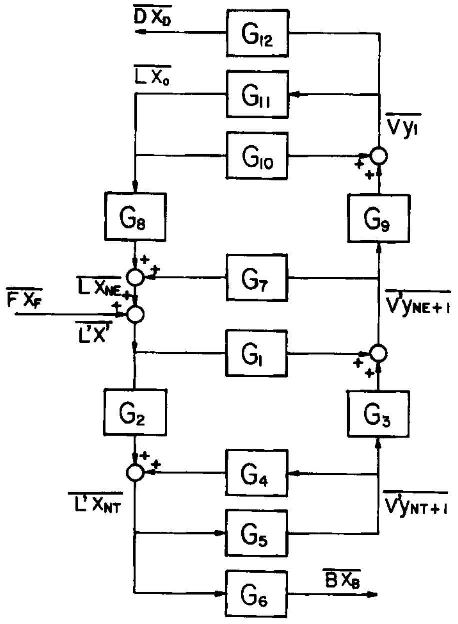

Figure 5. Block diagram of a distillation column.

Table 2. Gains for Towers Studied

Case	α	$(1 - x_D) = x_B$	G_4G_5	$G_{10}G_{11}$	H_EH_S
1	4	0.05	0.813	0.113	0.256
2	4	0.001	0.803	0.120	0.649
3	4	0.00001	0.517	0.110	0.948
4	2	0.05	0.767	0.406	0.724
5	2	0.01	0.603	0.563	0.985
6	2	0.00001	0.561	0.383	0.99986

gain does. This is particularly true for the three towers where the linear analysis is in error. Thus, one can conclude that the proximity of the H_EH_S loop gain to 1.0 is the principle cause of the problem. In the following section the behavior of H_EH_S loop gain at various feed compositions is studied. From the insight obtained from loop gain behavior, an improved method of estimating tower time constants is proposed.

Behavior of Gain of H_E and H_S for Towers Studied

The symbols K_E and K_S are used for the gain of H_E and H_S, respectively. Figure 6 plots K_EK_S loop gain at various feed compositions for cases 3, 5, and 6. These plots were calculated by first designing a tower for a given steady state. Then x_F was changed holding the number of trays and flow rates fixed. The resulting K_EK_S loop gains were evaluated and plotted against x_F, using a modified version of Luyben's stepping technique program.

Figure 6 shows that K_EK_S changes in a highly nonlinear manner with x_F. The K_EK_S gain is close to 1.0 only in a very small region near the design value of x_F. By analogy with the simple recycle model analyzed earlier, we expect that the dominant time constant will change in a similar manner with recycle. Therefore, in the region where K_EK_S approaches 1.0 we expect the tower's dominant time constant to be large. However, outside this small region the tower's dominant time constant will be much smaller. By analogy with the simple recycle model we also expect the initial rate of response to a step in x_F to be independent of the time constant. Thus, even though a linear analysis predicts a large time constant close to design steady state, a tower's rate of response in this region can be fast. For the simple recycle system the rate of response is given by K/θ, and both K and θ are large but their ratio is not. The same effect occurs for towers. For the transients shown in Figure 3, large feed composition changes were used to force the towers; therefore, the towers operated at x_F values substantially different from the design value of x_F. This result suggests that one should use a perturbed value of x_F to estimate effective tower time constants. This approach is discussed in the following section.

Before using this analysis to develop improved time constants. It is worthwhile to reflect on the high-purity tower results. The mathematical analysis shows that the linear approximation is good in only a small region about design conditions. Also, the unusual circumstance exists that the gain changes rapidly at the boundary of this region and is nearly constant at a much lower value on each side of the region. A physical explanation is suggested by these results. In a very small region around the design conditions a small change in feed composition requires a large change in tower concentration profile. This is because the tower is changing from a condition in which each component is separated into two pure products to a condition in which one product is becoming very impure. All incremental feed component goes into the impure product. After this transition, one product is essentially pure and the other is impure. Further changes in feed composition require only a small change in tower composition profile. The linear analysis indicates a long time constant because making large changes in a tower's composition profile would take a long time. The fallacy is that this real effect is only true for a very narrow region around design.

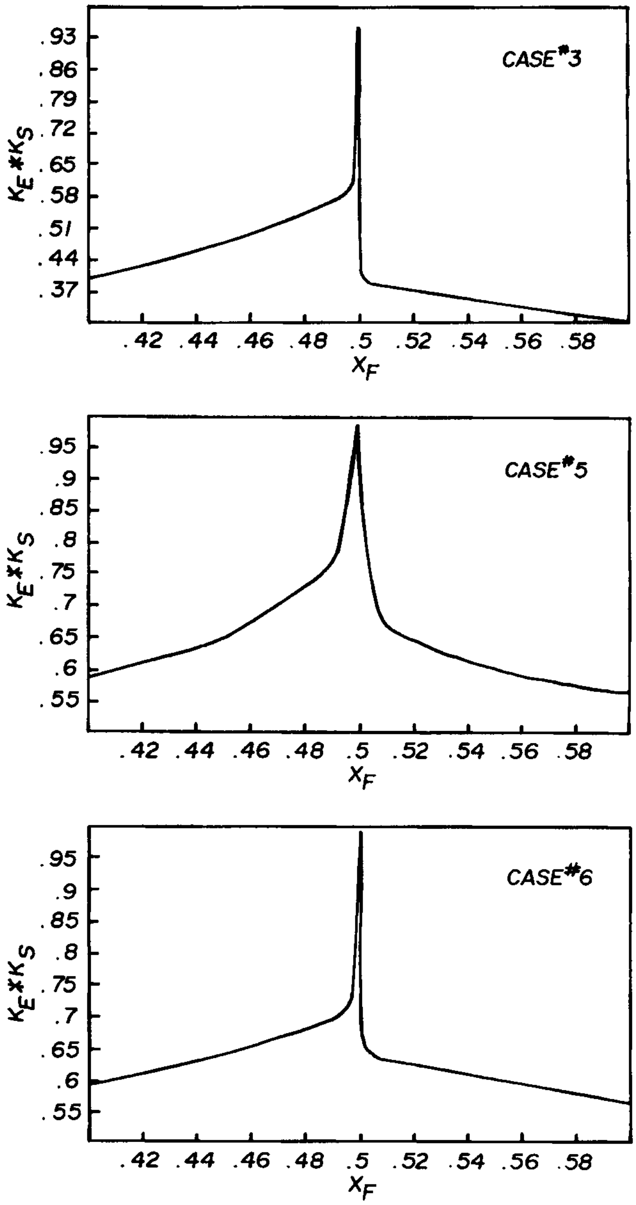

Figure 6. K_EK_S vs. x_F graph.
(a) Case 3. (b) Case 5. (c) Case 6.

An Improved Approach to Estimating Tower Time Constants

To test the hypothesis of using a perturbed steady state rather than the design steady state to estimate time constants, the following calculations were carried out. Using Luyben's (1973) approach, steady state conditions were calculated for x_F = 0.4

Table 3. Estimates of Tower Time Constants and Dead Times

			$x_F = 0.6$		$x_F = 0.4$	
Case	α	$(x_B = 1 - x_D)$	θ	τ	θ	τ
1	4	0.05	7.8	7.3	12.0	7.3
2	4	0.001	20.0	17.8	10.0	5.0
3	4	0.00001	22.2	27.0	8.8	9.1
4	2	0.05	30.0	14.0	22.0	11.6
5	2	0.001	28.5	22.0	20.0	13.0
6	2	0.00001	27.0	31.0	22.2	20.5

θ = time constant
τ = dead time

and 0.6 for all six towers shown in Table 1. The frequency response of x_D to changes in x_F at the perturbed steady state was calculated from the stepping technique. Finally, time constants were estimated from the −3db frequencies, which is the same method Fuentes and Luyben used to estimate the time constants in Table 1. The results of these calculations are shown in Table 3.

If the time constants shown in Table 3 are compared to the simulated tower transient response shown in Figure 3, it can be seen that they are reasonably accurate. In considering Figure 3 it should be noted that the y axis is a logarithmic scale to show the large composition changes that occur. The time constants in Table 3 are considerably more accurate than those shown in Table 1. An interesting effect that occurs when a perturbed steady state is used is that the phase angle is greater than −45° at the −3db frequency. This increase in phase angle does not occur when the design steady state is used for the stepping technique. If the increase in phase angle over −45° is attributed to a dead time in the system then the dead times shown in Table 3 can be calculated. If one compares the dead times in Table 3 to those shown in Figure 3, it can be seen that the agreement is very good. The time constants and dead time calculated at x_F values of 0.4 are smaller than those calculated for x_F values of 0.6. The results shown in Figure 3 indicate that the response is indeed faster when x_F drops than when it increases, particularly for the highest purity towers.

To check whether or not the proximity of the K_EK_S gain to 1.0 only occurs for x_F values near 0.5 and symmetric product splits, $x_B = 1 - x_D$, a number of other high-purity towers were studied. The properties of these additional towers are shown in Table 4. It was found that in almost all cases the K_EK_S gain was close to 1.0 at design conditions, but that it changed sharply with changes in x_F. As can be seen in Table 4, time constants estimated from design conditions are typically large. An exception occurs for all but one $x_F = 0.75$ case where the time constants obtained from the normal and perturbed states are in rough agreement. Thus, the effect of the K_EK_S gain on tower time constant estimates appears to be a very common effect for distillation towers. It can be noted that the effect of the K_EK_S gain will also be seen in every linearized model involving the tower. This is true since the tower itself involves the positive feedback loop and the loop is always operational. We have calculated frequency responses and estimated time constants for forcing variables other than feed composition, e.g., reflux flow, distillate flow, vapor flow, etc. The same positive feedback gain effects occur in these cases as occur when x_F is the forcing variable. To develop transfer functions for other forcing variables one should linearize around a perturbed steady state. The approach used in this paper can be easily extended to real industrial towers. To carry out this extension one needs a design program that can be used to calculate a perturbed steady state in the tower. Further, one needs to linearize the dynamic equations for one tray and then use Luyben's stepping approach to calculate frequency responses numerically. Finally, the effect of K_EK_S gain is associated with the fact that high-purity products are produced at both ends of a tower. If such a state occurs in a tower, even though it is not the design state, then a linear analysis will predict a large time constant. For example one could produce a high-purity steady state by canceling the effect of a disturbance through some type of control action.

The effect of the K_EK_S gain is important in terms of short-cut methods of estimating tower time constants. In developing such

Table 4. Estimates of Tower Time Constants and Dead Times

						$\Delta x_F = +0.1$ Perturbed		$\Delta x_F = -0.1$ Perturbed		Linear
Case	α	N_T	x_F	x_D	x_B	θ	τ	θ	τ	θ
1	4	26	0.5	0.99	0.001	22.2	13.0	8.10	5.7	45
2	4	26	0.5	0.999	0.01	28.01	24.0	8.84	8.5	140
3	4	26	0.25	0.99	0.001	16.13	16.9	14.03	4.9	126
4	4	26	0.25	0.999	0.01	16.10	23.9	14.28	6.2	1,176
5	4	26	0.75	0.99	0.001	12.50	7.6	6.33	4.9	13
6	4	26	0.75	0.999	0.01	20.00	14.6	6.90	6.6	25
7	2	40	0.5	0.99	0.001	29.12	18.7	21.79	13.7	1,000
8	2	40	0.5	0.999	0.01	39.80	34.7	22.20	22.2	444
9	2	40	0.25	0.99	0.001	21.15	22.1	33.33	11.6	398
10	2	40	0.25	0.999	0.01	24.39	31.9	33.33	17.4	3,984
11	2	60	0.25	0.99999	0.00001	12.59	23.3	31.64	13.2	558,000
12	2	40	0.75	0.99	0.001	19.23	12.4	17.24	12.0	33
13	2	40	0.75	0.999	0.01	26.32	20.6	18.18	17.4	50
14	2	60	0.75	0.99999	0.00001	12.50	15.4	14.03	10.2	352,000

θ = time constant
τ = dead time

methods one should consider perturbed rather than steady state operating points. It can be noted that Wahl and Harriott's (1970) results were calculated by considering the design steady state only. One approach to a shortcut method is to develop transfer function approximations for each of the G_i's in Figure 5. Then the method of moments can be used to approximate the resulting Eqs. 9 and 10 as first-order dead-time transfer functions. We are currently investigating this approach and will report our results in a later publication.

Acknowledgment

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Notation

B = bottoms flow
D = distillate flow
F = feed flow
G_i = transfer function
H_E = transfer function for enriching section
H_s = transfer function for stripping section
K = moment approximation gain
K_E = gain for H_E
K_F = forward path gain
K_R = reverse path gain
K_s = gain for H_s
L = reflux flow
L' = liquid flow below feed tray
N_E = trays in enriching section
N_s = trays in stripping section
N_T = trays in entire tower
s = Laplace transform variable
T_F = forward path time constant
T_R = reverse path time constant
V = vapor flow
x = input
x_B = bottoms mole fraction
x_F = feed mole fraction
x_D = distillate mole fraction
x_{NE} = mole fraction of liquid from rectifying section
x' = mole fraction of liquid from feed tray
y = output
y_{NE+1} = mole fraction of vapor from stripping section

Greek letters

α = relative volatility
θ = approximate time constant
τ = dead time

Superscripts

$\bar{\ }$ = Laplace transformation

Appendix

It is straightforward to develop bounds for the G_4G_5 and $G_{10}G_{11}$ gains for high-purity towers. The symbol K_i is used for the gain of G_i. The G_5 transfer function is considered first. A material balance on the reboiler gives

$$L'x_{NT} = Vy_{NT+1} + Bx_B \tag{A1}$$

Assuming equilibrium between y_{NT+1} and x_B gives

$$y_{NT+1} = Kx_B \tag{A2}$$

Since bottoms composition is small

$$K \simeq \alpha \tag{A3}$$

Substitution of Eqs. A2 and A3 into Eq. A1 gives

$$\left(V + \frac{B}{\alpha}\right) y_{NT+1} = L'x_{NT} \tag{A4}$$

From Eq. A4, K_5 can be calculated as

$$K_5 = \frac{V}{V + B/\alpha} \tag{A5}$$

An upper bound on K_4 can be calculated by considering the following material balance

$$x_F F + Vy_{NT+1} = L'x_{NT} + Dx_D \tag{A6}$$

If $y_{NT+1}\,V$ increases and the D stream is almost pure

$$\frac{d(Dx_D)}{d(y_{NT+1}\,V)} \rightarrow 0. \tag{A7}$$

Further to calculate K_4, Fx_F is held constant. Substitution of Eq. A7 into A6 and differentiation with respect to $L'x_{NT}$ give

$$\frac{d(L'x_{NT})}{d(y_{NT+1}\,V)} = K_4 \rightarrow 1.0 \tag{A8}$$

If Vy_{NT+1} decreases, then the change in Dx_D has to be taken into account in calculating K_4. It is straightforward to show that K_4 must be less than 1.0 if the material balance, Eq. A6, is satisfied. Thus, an upper bound on K_4 is 1.0. As a result

$$K_4K_5 < \frac{V}{V + B/\alpha} \tag{A9}$$

To develop bounds on $K_{10}K_{11}$ consider G_{11} first. If a total condenser is used, a material balance on the condenser gives

$$Vy_1 = Dx_D + Lx_D \tag{A10}$$

Differentiating Eq. A10 gives K_{11} as

$$K_{11} = \frac{L}{V} = \frac{L/D}{1 + L/D} \tag{A11}$$

The same approach that showed that $K_4 < 1.0$ can be used to show that $K_{10} < 1.0$. Thus, an upper bound on $K_{10}K_{11}$ is

$$K_{10}K_{11} < \frac{L/D}{1 + L/D} \tag{A12}$$

Equations A9 and A12 can be used to determine for which towers the G_4G_5 and $G_{10}G_{11}$ loops may cause trouble. These towers are those where α is large, B is small, and reflux ratios are large. In most towers K_4K_5 and $K_{10}K_{11}$ will not approach 1.0 very closely. For example, to have $K_{10}K_{11}$ be 0.98, Eq. A12 shows that the reflux ratio must be greater than 49. Similar estimates can be calculated from Eq. A9. However, a value of K_4K_5 or $K_{10}K_{11}$ of say 0.9 can indicate a prolonged transient. Consider the $K_{10}K_{11}$ case. If a disturbance is such as to force K_{10} toward 1.0, then the $G_{10}G_{11}$ loop will respond slowly, as Figure 2 shows. As for time constant estimates being in error by orders of magnitude, Eqs. A9 and A12 indicate that this effect is not due to G_4G_5 or $G_{10}G_{11}$. When such large errors occur they are due to recycle loops within the tower itself.

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